

Quantum bit commitment with cheat sensitive binding and approximate sealing

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Abstract. This paper proposes a cheat sensitive quantum bit commitment (CSQBC) scheme based on single photons, in which Alice commits a bit to Bob. Here, Bob only can cheat the committed bit with probability close to 0 with the increasing of used single photons' amount. And if Alice altered her committed bit after commitment phase, she will be detected with probability close to 1 with the increasing of used single photons' amount. The scheme is easy to be realized with nowadays technology.

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1. Introduction

Bit commitment (BC) is a cryptographic task between two participants, which has a lot of applications to crucial cryptographic protocols including interactive zero-knowledge proof [1, 2, 3, 4], coin flipping [5, 6, 7], oblivious transfer [8, 9], multiparty secure computation [10, 11, 12, 13], and so on.

Generally, BC mainly consists of two phases, *commitment* phase and *opening* phase. In *commitment* phase, Alice chooses a bit b ($b = 0$ or 1) which she wants to commit to Bob, and gives him some encrypted information about the bit, which can not be decrypted by him before *opening* phase. Later, in *opening* phase, Alice announces some information for decrypting b and the value of b . After decryption, Bob obtains an output b' . The commitment would be accepted by Bob if $b' = b$. Otherwise, the commitment would be rejected if $b' \neq b$. Bit commitment must meet the following needs: *Correctness*. Bob should always accept with $b' = b$ if both participants are honest. *Sealing*. Before opening phase, Bob can not know b . *Binding*. Alice can not change b 's value after the commitment phase.

There are several quantum approaches [5, 14] have been considered to guarantee the unconditional security of quantum BC (QBC) protocols, such as quantum key distribution (QKD) protocols [15, 16, 17]. Unfortunately, it was concluded that unconditionally secure QBC can never be achieved in principle, which was referred to as the Mayers-Lo-Chau (MLC) no-go theorem [18, 19, 20]. Although unconditional secure QBC protocols are not existent, there are several schemes satisfying special security models, such as cheat sensitive protocol, relativistic protocol, have been proposed [21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31]. Among them, an important class is cheat sensitive QBC (CSQBC) which is proposed by L. Hardy and A. Kent [21] first. In CSQBC, assuming that the commitment will eventually be opened, Bob cannot alter the committed bit after the commitment phase without risking Bob's detection, and Alice cannot extract information about the committed bit before the opening phase without risking Bob's detection as well. In other words, cheat sensitivity means that all the cheat strategies should be detected with nonzero probability in the protocol.

In this paper, we propose a variant CSQBC scheme based on single photons. In the scheme, cheat sensitive is one-way, which is only available in binding. If Alice alters her committed bit, she will be detected with probability close to 1 with the amount's increasing of used single photons. As for sealing, Bob only can cheat the committed bit with probability $\frac{1}{2} + \varepsilon$, where ε is close to 0 with the amount's increasing of used single photons. When $\varepsilon = 0$, the one-way CSQBC is more secure than the two-ways CSQBC as the full sealing is more secure than cheat sensitive sealing. However, since MLC no-go theorem said $\varepsilon = 0$ is impossible, we only could search for $\varepsilon \rightarrow 0$ in one-way CSQBC.

This paper is organized as follows. Sec. II shows the one-way CSQBC scheme. In Sec. III, we prove that the scheme is cheat sensitive in binding and approximate sealing. And the protocol's practicability is also analyzed. Finally, Sec. IV is a short conclusion.

2. The Quantum Bit Commitment Scheme

In this protocol, Alice will commit a bit b to Bob. Single photons will be used by them, each of which is prepared as one of the four states $\{|0\rangle, |1\rangle, |+\rangle, |-\rangle\}$ randomly where $|0\rangle$ and $|1\rangle$ are the two eigenstates of the Pauli operator σ_z , $|+\rangle$ and $|-\rangle$ are the two eigenstates of the Pauli operator σ_x . For the cheat sensitive in binding and approximate sealing, error correcting code (ECC) will be used here. The specific steps of the protocol are described as follows:

[Pre-commitment phase]

(1) Alice and Bob agree on a ECC (n, k, d) -code C [32], which uses n bits codeword to encode k bits word, and the distance between any two codewords is d .

(2) Alice chooses a nonzero random n -bit string $r = (r_1, r_2, \dots, r_n)$ where $r_i \in \{0, 1\}$ and announces it to Bob. Alice uses it to divide all the n -bit codeword $c = (c_1 c_2 \dots c_n)$ in C into two subsets $C_{(0)} \equiv \{c \in C | c \odot r = 0\}$ and $C_{(1)} \equiv \{c \in C | c \odot r = 1\}$, where $c \odot r \equiv \bigoplus_{i=1}^n c_i \wedge r_i$.

(3) Bob prepares an ordered n photons sequence $s = (s_1, s_2, \dots, s_n)$, in which each s_i is randomly in one of the four states $(|0\rangle, |1\rangle, |+\rangle, |-\rangle)$. Then Bob sends the photons sequence s to Alice.

[Commitment phase]

(4) According to the commitment bit b , Alice chooses a codeword c from $C_{(b)}$ randomly.

(5) When $c_i = 0$, Alice measures the i -th photon s_i in basis Z . Else when $c_i = 1$, Alice measures the i -th photon s_i in basis X . Then she obtains the outcomes $o = (o_1, o_2, \dots, o_n)$, where $o_i \in \{|0\rangle, |1\rangle, |+\rangle, |-\rangle\}$.

(6) When $o_i \in \{|0\rangle, |+\rangle\}$, Alice sets $o'_i = 0$. When $o_i \in \{|1\rangle, |-\rangle\}$, Alice sets $o'_i = 1$. Then Alice announces $o' = (o'_1, o'_2, \dots, o'_t)$ to Bob.

[Opening phase]

(7) Alice announces committed bit b , o and c to Bob.

(8) Bob checks whether o is right or not. The rule is that when $o'_i = 0$ (or 1), it should be $o_i \in \{|0\rangle, |+\rangle\}$ (or $\{|1\rangle, |-\rangle\}$). Then Bob checks whether $c \odot r = b$ or not. If both of them are right, he accepts the committed bit. Else, he rejects the committed bit.

3. Analysis

In the presented protocol, without considering the noise in the quantum channels and equipments, Bob will always accept Alice's committed bit as $c \odot r = b$ when both of them are honest.

However, as a quantum bit commitment protocol, Alice and Bob do not trust to each other, furthermore, one of them may be dishonest and perform cheat strategies. So we will analyze the scheme's security in the following two cases, (1) a dishonest Alice and an honest Bob, (2) a dishonest Bob and an honest Alice. Generally, the case

that neither Alice nor Bob is honest will not be considered since it will be a quantum gambling.

And the real-life setting will bring some troubles to the protocol. In this section, we will analyze the protocol's practicability following its security analysis.

3.1. Cheat sensitive binding

In the protocol, ECC (n, k, d) -code C is used, in which the distance between any two codewords is d . It means that Alice should change d bits in c if she wants to alter committed bit b to b' , where $b, b' \in \{0, 1\}$ and $b \neq b'$. Further, Alice could use the slyest strategy, in which she first commits a bit b'' other than 0 or 1, i.e., she choose a bit string c' which is contained in neither $C_{(0)}$ nor $C_{(1)}$, and let the Hamming distance between c' and any one of $C_{(0)}$ and $C_{(1)}$ be $d/2$. Then she only needs to change $d/2$ bits in c' to cheat $b = 0$ or $b = 1$.

When Alice announces o' , it means that she had committed something regardless whether she has measured the photons or not. In the opening phase, what she should do is to make o , and c tally with o' and her wanted b . For instance, if she wants to cheat $b = 0$, c should be one in the set $C_{(0)}$. We know that both of $o_i = |0\rangle$ and $|+\rangle$ ($o_i = |1\rangle$ and $|-\rangle$) are possible when $o'_i = 0$ (or $o'_i = 1$), so 2^n different o are legal corresponding to one o' . Then the cheat strategy degenerates to a simpler thing: Bob sends a photon in one of states $\{|0\rangle, |1\rangle, |+\rangle, |-\rangle\}$ to Alice. Alice could do anything on it, then she should say whether the state is in the set $\{|0\rangle, |+\rangle\}$ or $\{|1\rangle, |-\rangle\}$. If she is right, she could cheat successfully with probability 1 as the states in the set are always legal. But if she is wrong, her cheating will be detected with probability 1/2, as she can avoid to be detected when her announced basis is wrong but be detected with certainty when her announced basis is right.

Now we analyze how can Alice distinguish the single photon from the sets $\{|0\rangle, |+\rangle\}$ and $\{|1\rangle, |-\rangle\}$. Since the photon is always hold in Alice's hand, she would not use any ancilla states, but measure the photon directly. We suppose the measurement basis is $\{|r_0\rangle, |r_1\rangle\}$, where $|r_0\rangle = \cos\theta|0\rangle + \sin\theta|1\rangle$ and $|r_1\rangle = \sin\theta|0\rangle - \cos\theta|1\rangle$. It should be that

$$|0\rangle = \cos\theta|r_0\rangle + \sin\theta|r_1\rangle, \quad (1a)$$

$$|1\rangle = \sin\theta|r_0\rangle - \cos\theta|r_1\rangle, \quad (1b)$$

$$|+\rangle = \cos(\frac{\pi}{4} - \theta)|r_0\rangle - \sin(\frac{\pi}{4} - \theta)|r_1\rangle \quad (1c)$$

$$|-\rangle = \sin(\frac{\pi}{4} - \theta)|r_0\rangle + \cos(\frac{\pi}{4} - \theta)|r_1\rangle. \quad (1d)$$

When the photon is $|0\rangle$ or $|1\rangle$, Alice could distinguish the two sets successfully with probability $\cos^2\theta$. When the photon is $|+\rangle$ or $|-\rangle$, Alice would distinguish the two sets

successfully with probability $\cos^2(\frac{\pi}{4} - \theta)$. So the total probability of Alice distinguishes the two sets successfully is

$$\begin{aligned} P &= \frac{\cos^2 \theta + \cos^2(\frac{\pi}{4} - \theta)}{2} \\ &= \frac{2 + \sin 2\theta + \cos 2\theta}{4} \\ &= \frac{2 + \sqrt{2} \cos(\frac{\pi}{4} - 2\theta)}{4}. \end{aligned} \quad (2)$$

It should be that $\frac{2-\sqrt{2}}{4} \leq P \leq \frac{2+\sqrt{2}}{4}$. If Alice distinguishes them unsuccessfully, Bob will detect the cheating when his basis is same with what Alice announced. So Alice will be detected with at least probability $\frac{1-(\frac{2+\sqrt{2}}{4})}{2}$ when she cheated on one photon. As she must cheat on at least $d/2$ photons, she will be detected with probability $1 - (1 - \frac{1-(\frac{2+\sqrt{2}}{4})}{2})^{d/2} = 1 - (\frac{6+\sqrt{2}}{8})^{d/2}$ for altering the committed bit. With the increasing of d , the probability will be close to 1. Since d increases with the increasing of n normally, it means that Alice will be detected with probability close to 1 with the amount's increasing of used single photons if she alters the committed bit.

3.2. Approximate sealing

Before the opening phase, a dishonest Bob might cheat Alice's committed bit with the states he sent and Alice's announcement.

In fact, without any cheat strategies, a curious Bob could obtain some information about o_i . When the i th photon Bob sent is $|0\rangle$, if Alice said her measurement outcome is in the set $\{|0\rangle, |+\rangle\}$, he can guess the basis Alice used is Z . Else if Alice said her measurement outcome is in the set $\{|1\rangle, |-\rangle\}$, he can guess the basis Alice used is X . With this way, he will success with probability $3/4$ to obtain o_i before the opening phase. However, since the distance between any two code words in $C_{(0)}$ and $C_{(1)}$ is d , Bob must obtain more than $n - d$ bits to extract valid committed information. So Bob could cheat successfully with probability $(\frac{3}{4})^{n-d}$.

Bob has a more sufficient cheat strategy. Instead of sending a single photon to Alice, Bob could cheat by sending one participle of an entangle state to Alice. After she measured it, he measures his participle for analyzing o_i . The best thing to him is obtaining a same state as Alice's, i.e, obtaining a photon in state $|0\rangle$ (or $|1\rangle$, or $|+\rangle$, or $|-\rangle$), when $o_i = |0\rangle$ (or $|1\rangle$, or $|+\rangle$, or $|-\rangle$). Then according to o'_i , he calculates o_i and c_i . For these, he should measure the state which is in one of $\{|0\rangle, |+\rangle\}$ to make sure what state it is.

The problem of optimal state estimation has been studied in great detail previously[33], and in particular the optimal measurement for discriminating two density operators[34] is well known. Using the optimal measurement, the maximum probability that Bob estimates c_i is

$$P^{max} = \frac{1}{2} + \frac{1}{4} \text{Tr}|\rho_{|0\rangle} - \rho_{|+\rangle}| = \frac{1}{2} + \frac{\sqrt{2}}{4} \quad (3)$$

, where $|\rho_{|0\rangle} - \rho_{|+\rangle}| = \sqrt{(\rho_{|0\rangle} - \rho_{|+\rangle})^\dagger (\rho_{|0\rangle} - \rho_{|+\rangle})}$, and $(\rho_{|0\rangle} - \rho_{|+\rangle})^\dagger$ is Hermitian conjugate or adjoint of the $(\rho_{|0\rangle} - \rho_{|+\rangle})$ matrix.

So Bob could obtain c_i with success probability $\frac{1}{2} + \frac{\sqrt{2}}{4}$. Since the distance between any two code words is d , Bob should know more than $n - d$ bits to obtain valid information. The probability of this case is $(\frac{1}{2} + \frac{\sqrt{2}}{4})^{n-d}$. Namely, Bob only can cheat the committed bit with probability $\frac{1}{2} + \varepsilon$, where ε is close to 0 with the increasing of $n - d$. When $\varepsilon \rightarrow 0$, Bob's cheat strategy almost likes guessing. Since $n - d$ increases with the increasing of n normally, it means that Bob only can cheat the committed bit with probability close to 0 with the increasing of used single photons' amount.

3.3. Practicability

In the presented protocol, only BB84 states, X and Y bases measurements are used, all of which can be implemented with nowadays technology. In QBC, the period between commitment phase and opening phase may be very long. If quantum states are needed to be stored during this period, the protocol will be difficult to realize with nowadays technology. Here, quantum storages are not needed in the proposed QBC. So compared with some protocols in which long-time quantum memories are used, our protocol is more practicable.

Multi-photon is an important problem which has brought some troubles to practical quantum protocols. Now we analyze its effect to the presented QBC. We first consider the case happened in i th order. When Bob sends a pulse containing two photons, Alice should measure one photon in basis X , the other in basis Z . If the two outcomes happen to be $\{|0\rangle, |+\rangle\}$ or $\{|1\rangle, |-\rangle\}$, she can cheat to $c_i = 0$ and $c_i = 1$ easily by announcing $o'_i = 0$ or $o'_i = 1$ at step (6) and announce her wanted c_i at step (7). However, if the two outcomes happen to be $\{|0\rangle, |-\rangle\}$ or $\{|1\rangle, |+\rangle\}$, Alice can not perform this cheating. Namely, to one multi-photon, she could perform the cheating with probability $1/2$. For cheating successfully, Alice needs to change $d/2$ bits in c at least. When the multi-photon rate η_m is less than $\frac{d}{2} \times 2 \times \frac{1}{n} = \frac{d}{n}$, she could not cheat successfully. So Bob should set the multi-photon rate of his source as a small enough value for secure.

The loss and error appearing in quantum channels and devices are another important problems in practical quantum protocols. Here, Alice could said some pulse which contains only one photon is lost. Then she has more chances to cheat. She also could say some of the attacked bit as error bit. So the loss rate η_l and error rate η_e could not be too large. It should be $\frac{\eta_m}{2} + \eta_l + \eta_e \ll \frac{d}{2n}$.

4. Conclusion

To summarize, in this paper, we have dealt with a quantum bit commitment protocol based on single photons. In our scheme, Alice commits a value by performing some measurements on the single photons which are sent from Bob. With the increasing of photons' amount, Bob only can cheat the committed bit with probability close to 0. On the other hand, if Alice alters her committed bit after commitment phase, she will be detected with probability close to 1 with the increasing of photons' amount. It is easy

to be realized with nowadays technology.

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- [1] Brassard, G., Chaum, D., Crépeau, C.: Minimum disclosure proofs of knowledge. *Journal of Computer and System Sciences*, 37, 156-189 (1988)
- [2] Goldwasser, S., Micali, S., Rackoff, C.: The knowledge complexity of interactive proof-systems. In *STOC 85*, 291-304 (1985) (1989)
- [3] Goldreich, O., Micali, S., Wigderson, A.: Proofs that yield nothing but their validity or all languages in NP have zero-knowledge proof systems. *Journal of the ACM* 38(1), 691-729 (1991)
- [4] Nascimento, J.C.D., Ramos, R.V.: Quantum protocols for zero-knowledge systems. *Quant. Inf. Proc.* 9 (1), 37-46 (2010)
- [5] Brassard, G., Crépeau, C.: Quantum bit commitment and coin tossing protocols. in *Advances in Cryptology: Proceedings of Crypto90, Lecture Notes in Computer Science Vol. 537* (Springer-Verlag, Berlin), 49-61 (1991)
- [6] Nayak, A., Shor, P.: Bit-commitment-based quantum coin flipping. *Phys. Rev. A* 67, 012304 (2003)
- [7] Silman, J., Chailloux, A., Aharon, N., Kerenidis, I., Pironio, S., Massar, S.: Fully Distrustful Quantum Bit Commitment and Coin Flipping. *Phys. Rev. Lett.* 106, 220501 (2011)
- [8] Bennett, C.H., Brassard, G., Crépeau, C., Skubiszewska, M. H.: Practical quantum oblivious transfer protocols. in *Advances in Cryptology: Proceedings of Crypto91, Lecture Notes in Computer Science Vol. 576* (Springer-Verlag), 351-366 (1992)
- [9] He, G. P., Wang, Z.D.: Oblivious transfer using quantum entanglement. *Phys. Rev. A* 73, 012331 (2006)
- [10] Li, Y.B., Wen, Q.Y., Qin, S.J.: Comment on "Secure multipartycomputation with a dishonest majority via quantum means". *Phys. Rev. A* 84, 016301 (2011)
- [11] Li, Y.B., Wen, Q.Y., Gao, F., Jia, H.Y., Sun, Y.: Information leak in Liu et al.'s quantum private comparison and a new protocol. *Eur. Phys. J. D* 66, 110-115 (2012)
- [12] Yang, Y.G., Wen, Q.Y.: An efficient two-party quantum private comparison protocol with decoy photons and two-photon entanglement. *J. Phys. A Math. Theor.* 42, 055305 (2009)
- [13] Li, Y.B., Qin, S.J., Yuan, Z., Huang, W., Sun, Y.: Quantum private comparison against decoherence noise. *Quant. Inf. Proc.* 12 (6), 2191-2205 (2013)
- [14] Brassard, G., Crépeau, C., Jozsa, R., Langlois, D.: A quantum bit commitment scheme provably unbreakable by both parties. in *Proceedings of the 34th Annual IEEE Symposium on Foundations of Computer Science*, (IEEE, Los Alamitos), pp.362-371 (1993)
- [15] Bennett, C.H., Brassard, G.: Quantum cryptography: Public key distribution and coin tossing. in *Proceedings of IEEE International Conference on Computers, Systems, and Signal Processing*, Bangalore, India (IEEE, New York) pp. 17-179 (1984)
- [16] Lo, H.K., Chau, H.F.: Unconditional Security of Quantum Key Distribution over Arbitrarily Long Distances. *Science* 283, 2050 (1999)
- [17] Allati, A.E., Baz, M.E., Hassouni Y.: Quantum key distribution via tripartite coherent states. *Quant. Inf. Proc.* 10 (5), 589-602 (2011)
- [18] Mayers, D.: Unconditionally Secure Quantum Bit Commitment is Impossible. *Phys. Rev. Lett.* 78, 3414 (1997)
- [19] Lo, H. K., Chau, H.F.: Is quantum bit commitment really possible?. *Phys. Rev. Lett.* 78, 3410 (1997)
- [20] Li, Q., Li, C.Q., Long, D.Y., Chan, W.H., WuOn, C.H.: The impossibility of non-static quantum bit commitment between two parties. *Quant. Inf. Proc.* 11 (2), 519-527 (2012)
- [21] Hardy, L., Kent, A.: Cheat sensitive quantum bit commitment. *Phys. Rev. Lett.* 92, 157901 (2004)
- [22] Shimizu, K., Fukasaka, H., Tamaki, K., Imoto, N.: Cheat-sensitive commitment of a classical bit coded in a block of $m+n$ round-trip qubits. *Phys. Rev. A* 84, 022308 (2011)
- [23] Halvorson, H.: Remote preparation of arbitrary ensembles and quantum bit commitment. *J. Math. Phys.* 45 4920 (2004)
- [24] He, G.P.: Secure quantum bit commitment against empty promises. *Phys. Rev. A* 74, 022332 (2006)
- [25] Choi, J.w., Hong, D., Chang, K.Y., Chi, D.P., Lee, S.: Non-static Quantum Bit Commitment.

- arXiv:quant-ph/0901.1178
- [26] Wolf, S., Wullschleger, J.: Bit Commitment from Weak Non-Locality. arXiv:quant-ph/0508233
 - [27] He, G.P., Wang, Z.D.: Practically secure quantum bit commitment based on quantum seals. arXiv:quant-ph/0804.3531
 - [28] He, G. P.: Quantum key distribution based on orthogonal states allows secure quantum bit commitment. *J. Phys. A: Math. Theor.* 44, 445305 (2011)
 - [29] Adrian, K.: Unconditionally secure bit commitment with flying qudits. *New J. Phys.* 13, 113015 (2011)
 - [30] Yuen, H. P.: An unconditionally secure quantum bit commitment protocol. arXiv:1212.0938v1 (2012)
 - [31] Danan, A., Vaidman, Lev.: Practical quantum bit commitment protocol. *Quant. Inf. Proc.* 11 (3), 769-775 (2012)
 - [32] MacWilliams, F.J., Sloane, N.J.A.: *The Theory of Error-correcting Codes*. North-Holland Mathematical Lib (1977)
 - [33] Helstrom, C. W.: *Quantum detection and estimation theory*. Academic Press (New York) (1976)
 - [34] Fuchs, C. A.: Information Gain vs. State Disturbance in Quantum Theory. *Fortschr. Phys.* 46, pp 535-565 (1998)